Technical rate of substitution of an implicit function

If the equation $2Q^2 + 3LQ + L^2K^3 + 6K = 0$ implicitly defines a production function $Q = Q(K, L)$, find the marginal productivities. Additionally, indicate how much K must decrease if L is increased by one unit in order to keep the production constant.

Solution

First, we calculate the derivative from the implicit function:

$$Q'_K = -\frac{\partial f}{\partial K}\partial f \partial Q = -\frac{3L^2K^2 + 6}{4Q + 3L}$$

$$Q'_L = -\frac{\partial f}{\partial L}\partial f \partial Q = -\frac{3Q + 2LK^3}{4Q + 3L}$$

$$\text{TST}\left(\frac{K}{L}\right) = \frac{Q'_L}{Q'_K} = \frac{3Q + 2LK^3}{3L^2K^2 + 6}$$

K should decrease by $\frac{3Q+2LK^3}{3L^2K^2+6}$ if L is increased by one unit in order to keep the production constant.